Effects of Transducers on the Magnetic field of the tracker

Note: The magnetic field is being measured in Gauss, while the lengths have units of cm.

Introduction

Transducers are needed for communicating via optical fibers to the DAQ.Our tentative choice is the Stratos/Emerson RJS - ST31, shown in Figure 1. This is similar to the standard SFP transceiver but with a smaller footprint.

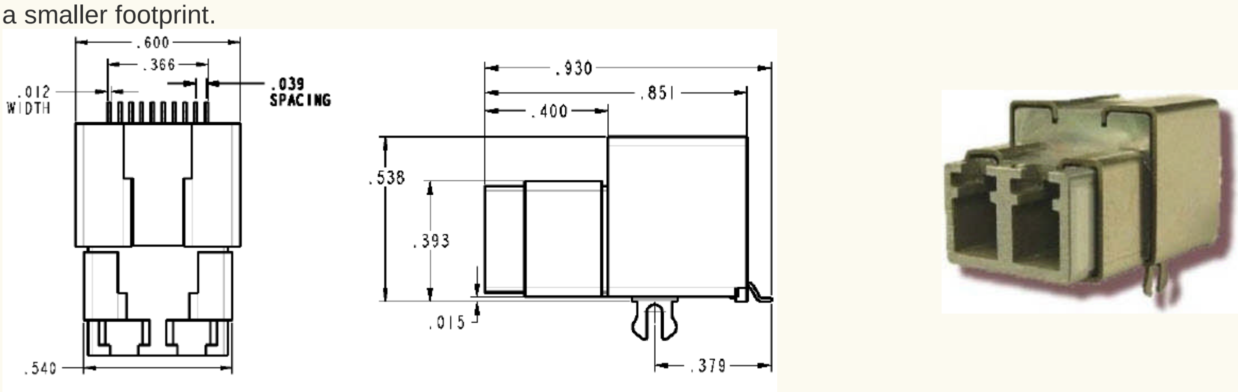


Figure 1. Stratos/Emerson RJS-ST31 optical transceiver. Dimensions in inches.

Unlike many other transducers, this one does not rely upon ferrites (which would saturate in our magnetic field). However it uses leaded, metal can packaging for the photo elements. The can and the leads are magnetic.
When sliding the Transducer into the magnetic field there could be seen an obvious change in the readings of the Hall probes. The transducer changed the field at 1cm distance by up to 15G.
Hence the transducer had to be analyzed more closely. I did this by taking measurements of the distortions at different positions of the transducer in relation to the Hall probes.

Is the transducer made up of two dipoles?

The first set of measurements I took, was by moving the Transducer in the x direction, having $z = y = 0$. Note figure 2 is drawn, such that we are in the $y = 0$ - plane.

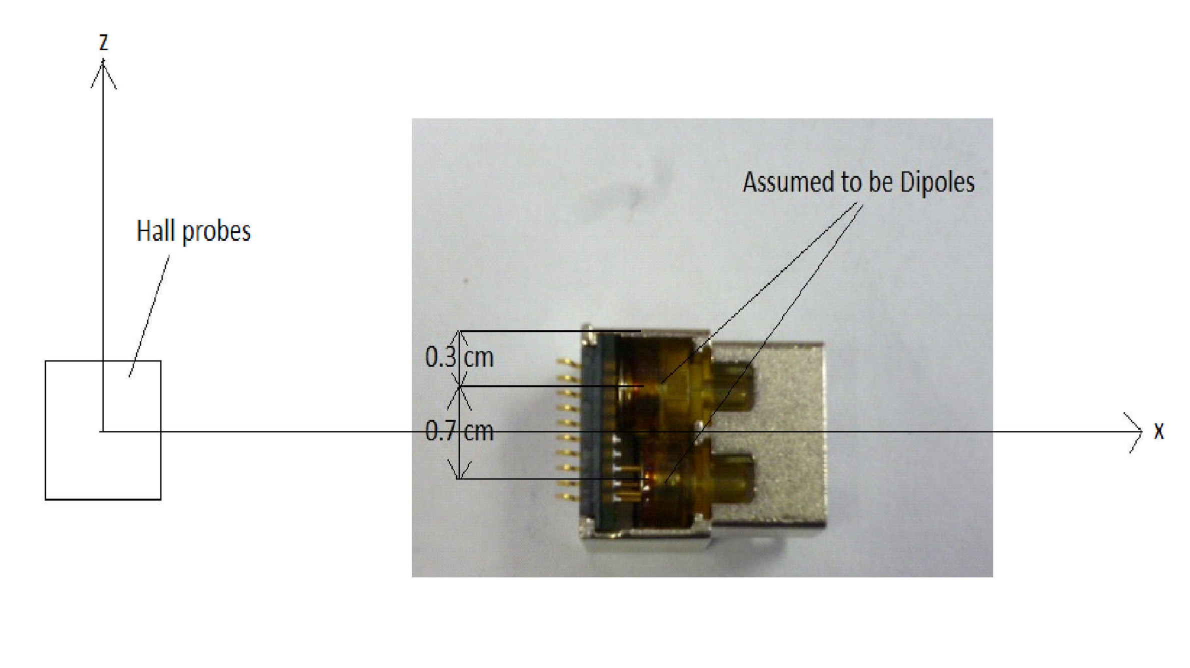


Figure 2: Sketch of the measurement situation with the y - axis going into the page.

We assumed that the two packagings for the photo elements would act as dipoles and therefore the 3D Hall probe would detect the sum of their magnetic fields.
The magnetic field obtained by a dipole orientated in the *z* - direction and positioned at the origin is given by :

$$\vec{B} = \left(\frac{3\,\mu 0\, m}{4\,\pi} \frac{z}{\sqrt{x^2+y^2+z^2}^{5}}\, x,\ \frac{3\,\mu 0\, m}{4\,\pi} \frac{z}{\sqrt{x^2+y^2+z^2}^{5}}\, y,\ \frac{\mu 0\, m}{4\,\pi} \left(\frac{-1}{\sqrt{x^2+y^2+z^2}^{3}} + \frac{3\, z^2}{\sqrt{x^2+y^2+z^2}^{5}} \right) \right)$$

When we consider the effect of the magnetic field of the upper dipole on the 3D Hall probe $y = 0$ and z = -0.7/2 cm = -0.35 cm therefore:

$$\vec{B}_u = \left(\frac{3\,\mu 0\, m}{4\,\pi} \frac{-0.35}{\sqrt{x^2+(-0.35)^2}^{5}}\, x,\ 0,\ \frac{\mu 0\, m}{4\,\pi} \left(\frac{-1}{\left(\sqrt{x^2+(-0.35)^2}\right)^{3}} + \frac{3\,(-0.35)^2}{\left(\sqrt{x^2+(-0.35)^2}\right)^{5}} \right) \right)$$

For the lower dipole we have $y = 0$ and z = 0.7/2 cm = 0.35 cm and hence:

$$\vec{B}_l = \left(\frac{3\,\mu 0\, m}{4\,\pi} \frac{0.35}{\sqrt{x^2 + (0.35)^2}^5} x, \right.$$

$$\left. 0, \frac{\mu 0\, m}{4\,\pi} \left(\frac{-1}{\left(\sqrt{x^2 + (-0.35)^2}\right)^3} + \frac{3\,(0.35)^2}{\left(\sqrt{x^2 + (0.35)^2}\right)^5} \right) \right)$$

Therefore the sum of the two fields, which is just the total field is given by:

$$\vec{B}_t = \left(0,\ 0,\ \frac{\mu 0\, m}{2\,\pi} \left(\frac{-1}{\left(\sqrt{x^2 + (-0.35)^2}\right)^3} + \frac{3\,(0.35)^2}{\left(\sqrt{x^2 + (0.35)^2}\right)^5} \right) \right) \qquad \Rightarrow$$

$$B_t = \frac{\mu 0\, m}{2\,\pi} \left(\frac{-1}{\left(\sqrt{x^2 + (-0.35)^2}\right)^3} + \frac{3\,(0.35)^2}{\left(\sqrt{x^2 + (0.35)^2}\right)^5} \right) = \frac{\mu 0\, m}{2\,\pi}\, p_1$$

I experimentally obtained results of $B_t$for different values of x and plotted B_t against p_1. This is being illustrated in Figure 3.

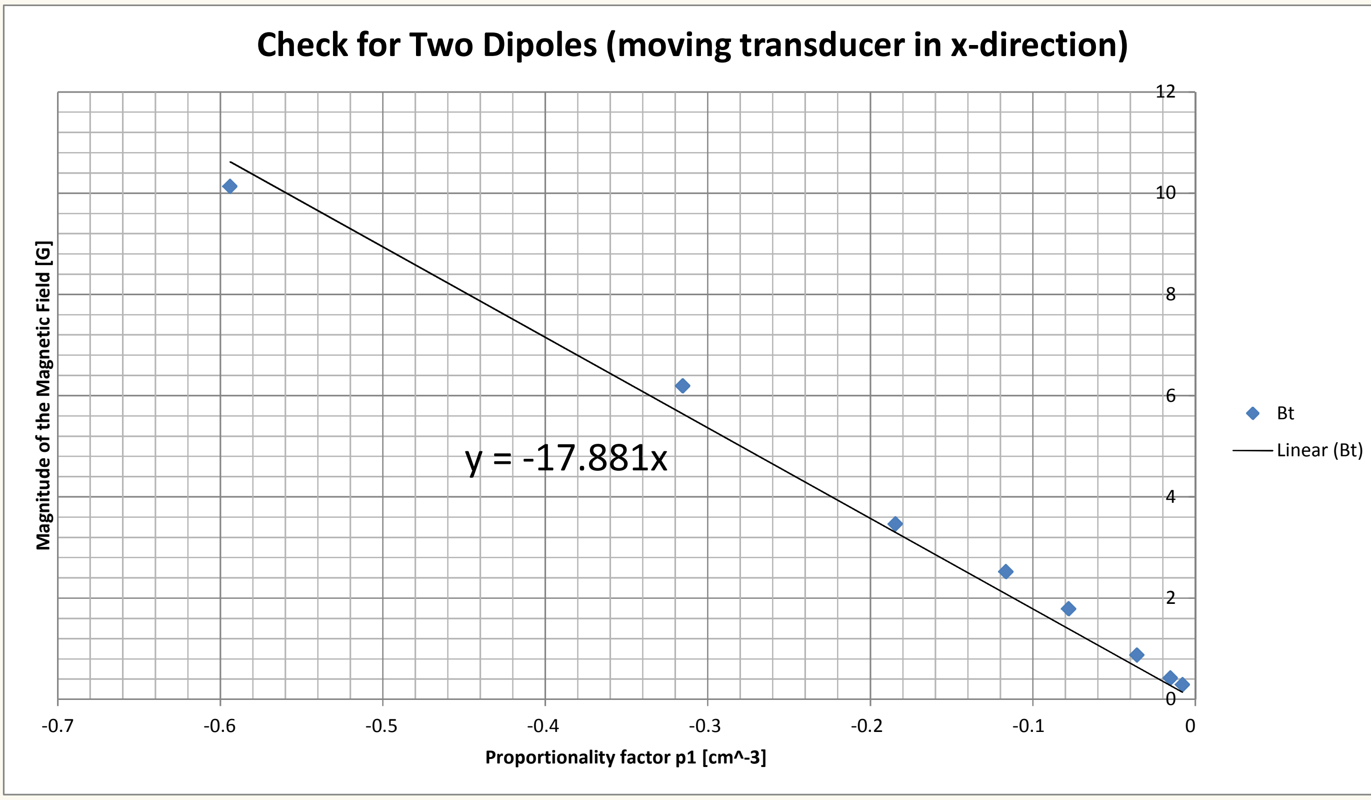


Figure 3: Check whether the transducer behaves as a combination of two dipoles when moving it in the x-direction

As I could easily apply a linear fit, I decided that the assumption, that the transducer is made up of two

dipoles is correct and from the slope of the graph I found an absolute value for $\frac{\mu 0\, m}{2 \pi}$, which is approximately 18.
In order to find out whether this value was sensible, I did a few other measurements of the distortions at different positions of the transducer in relation to the Hall probes.
This time I moved the transducer in the *y* - direction, while keeping *x* constant (*x*=1.6cm) and $z = 0$:

When we consider the effect of the magnetic field of the upper dipole on the 3D Hall probe $z =$ -0.7/2 cm = -0.35 cm therefore:

$$\vec{B}_u = \left(\frac{3\,\mu 0\, m}{4 \pi} \frac{-0.35}{\sqrt{(1.6)^2 + y^2 + (-0.35)^2}^5} 1.6,\ \frac{3\,\mu 0\, m}{4 \pi} \frac{-0.35}{\left(\sqrt{(1.6)^2 + y^2 + (-0.35)^2}\right)^5} y,\ \frac{\mu 0\, m}{4 \pi} \left(\frac{-1}{\left(\sqrt{(1.6)^2 + y^2 + (-0.35)^2}\right)^3} + \frac{3\,(-0.35)^2}{\left(\sqrt{(1.6)^2 + y^2 + (-0.35)^2}\right)^5} \right) \right)$$

For the lower dipole we have $z = 0.7/2$ cm = 0.35 cm and hence:

$$\vec{B}_l = \left(\frac{3\,\mu 0\, m}{4 \pi} \frac{0.35}{\sqrt{(1.6)^2 + y^2 + (0.35)^2}^5} 1.6,\ \frac{3\,\mu 0\, m}{4 \pi} \frac{0.35}{\left(\sqrt{(1.6)^2 + y^2 + (0.35)^2}\right)^5} y,\ \frac{\mu 0\, m}{4 \pi} \left(\frac{-1}{\left(\sqrt{(1.6)^2 + y^2 + (0.35)^2}\right)^3} + \frac{3\,(0.35)^2}{\left(\sqrt{(1.6)^2 + y^2 + (0.35)^2}\right)^5} \right) \right)$$

Therefore the sum of the two fields, which is just the total field is given by:

$$\vec{B}_t = \left(0,\ 0,\ \frac{\mu 0\, m}{2 \pi} \left(\frac{-1}{\left(\sqrt{(1.6)^2 + y^2 + (0.35)^2}\right)^3} + \frac{3\,(0.35)^2}{\left(\sqrt{(1.6)^2 + y^2 + (0.35)^2}\right)^5} \right) \right) \quad \Rightarrow$$

$$B_t = \frac{\mu 0\, m}{2 \pi} \left(\frac{-1}{\left(\sqrt{(1.6)^2 + y^2 + (0.35)^2}\right)^3} + \frac{3\,(0.35)^2}{\left(\sqrt{(1.6)^2 + y^2 + (0.35)^2}\right)^5} \right) = \frac{\mu 0\, m}{2 \pi} p_2$$

Similarly to above I experimentally obtained results of B_t for different values of *y* and plotted B_t against p_2:

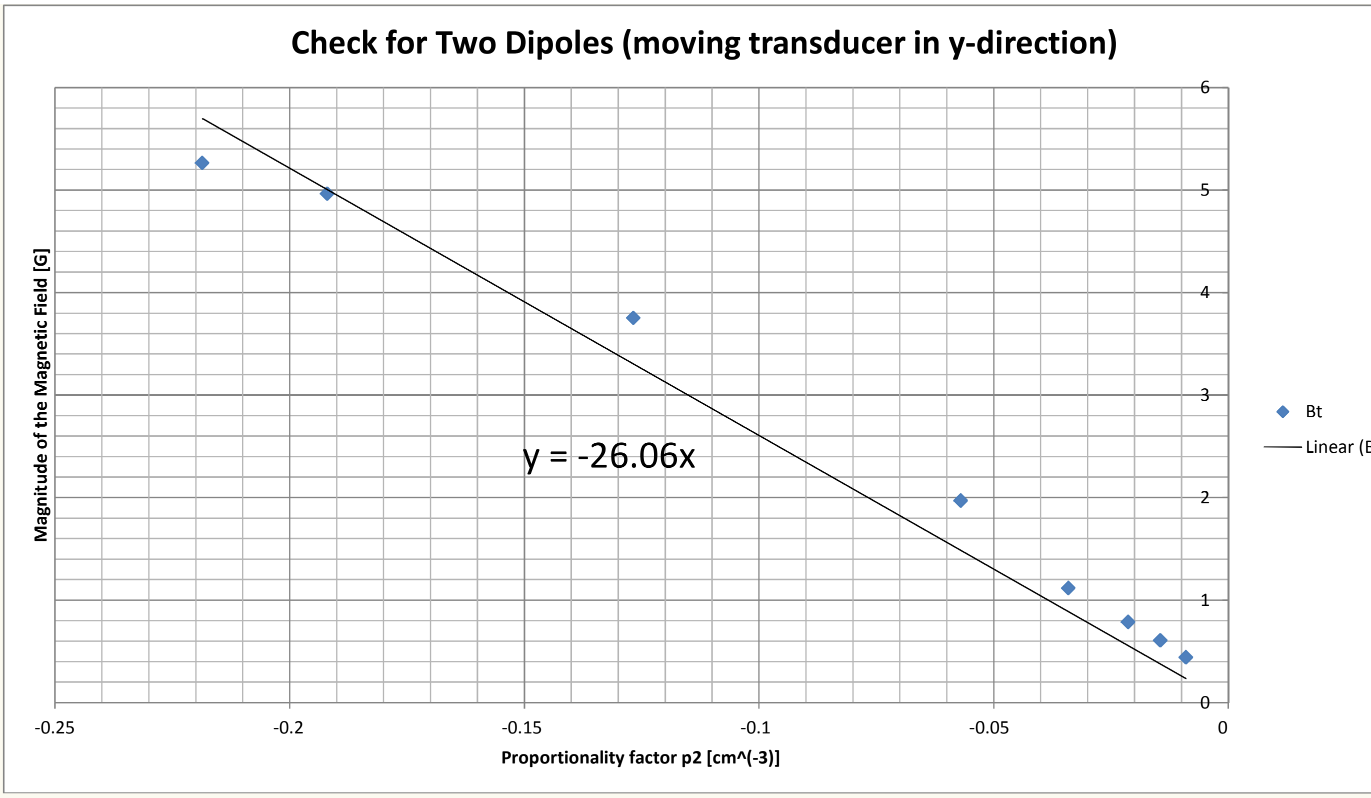


Figure 4: Check whether the transducer behaves as a combination of two dipoles when moving the transducer in the y-direction

Again I could apply a linear fit and obtained the value for $\frac{\mu 0\ m}{2\pi}$ from the slope of the graph. In this case this was approximately -26.

In the following part of this document I consider what happens to the magnetic field, when placing all the transducers into the tracker.

Calculation of the magnetic field, when taking all the transducers in the tracker into account

This code uses the infromation of the document 888.
We will position the origin at the centre of the tracker (i.e. in the centre of the circle when taking a slice of the tracker and inbetween the 9th and 10th station).
Each panel has two corresponding transducers which will be positioned directly next to each other in the FEE space 78cm from the *z*-axis.
The two dipoles within the transducer are approximately 0.7 cm apart, while the closest distance between dipoles of two corresponding pairs of transducers is 0.6.
We are interested in the field distortions due to the transducers in the volume 0cm $\leq r \leq$ 70cm.

```
Clear["`*"]
```

In[1]:=

```
c = μ0 m / (4 π); (*This just specifies what the constant
 c is (i.e. it specifies the properties of our dipole)*)
c = -20 / 2; (*Constant obtained experimentally when showing that the
 transducer behaves approximately like a combination of two dipoles*)
```

In[3]:=

```
Bcartesian[x_, y_, z_] :=
  {3 c z/Sqrt[x^2 + y^2 + z^2]^5 x, 3 c z/Sqrt[x^2 + y^2 + z^2]^5 y, c (-1/Sqrt[x^2 + y^2 + z^2]^3 + 3 z^2/Sqrt[x^2 + y^2 + z^2]^5)};
(*This is just a function giving the magnetic field obtained by a
  dipole orrientated in the z - direction and positioned at the origin*)
```

In[4]:=

```
ztr = {}; (* This just creates an empty list which will be used to store the z-
 coordinates of the transducers to the right of the origin*)
Do[z1 = 8.7 i - 3.45;
    z2 = 8.7 i - 1.25;
    z3 = 8.7 i + 1.25;
    z4 = 8.7 i + 3.45; (*z1 till z4 will give the z positions
   of the transducers positioned in the panels of a station*)
   r = Flatten [{Table[z1, {i, 1, 12}], Table[z2, {i, 1, 12}], Table[z3, {i, 1, 12}],
      Table[z4, {i, 1, 12}]}]; (*We create a list with 12 entries of z1,
  being followed by 12 entries of z2........followed by 12 entries of
   z4. We need 12 entries of each, because we have 3 panels in a plane,
  each needing 2 transducers with 2 dipoles*)
    ztr = Join[ztr, r];
    , {i, 1, 9}]; (*The Do loop just creates the z-
 coordinates of the transducers for the 9 stations to the right of the origin*)
ztl = -ztr; (*As we have a symmetric tracker,
this will just create a new list with the z-
 coordinates of the transducers for the 9 stations to the left of the origin*)
zt = Join[ztl, ztr]; (*This will create a list with all z-
 coordinates of the transducers, however notice that we first go
 from the origin to the left and then from the origin to the right*)
```

We will use cylindrical coordinates for now:
In order to find the closest distance of the 4 dipoles to the z-axis (i.e.r), we assume that they are positioned at x_1= -1, x_2 = -0.3, x_3 = 0.3 and x_4 = 1 (i.e. the position at which the transducers touch is at x = 0, the y - axis)

Hence we can determine r by $r_{1,4}= \sqrt{1^2 + 78^2} \approx 78.0064$ and $r_{2,3} = \sqrt{0.3^2 + 78^2} \approx 78.0006$

In[8]:=

```
rt = Flatten[Table[{78.0064, 78.0006, 78.0006, 78.0064}, {i, 1, Length[zt] / 4}]];
(*Creating the list with the r-coordinates for the transducers is
  simple because they will be put at the same place at each panel*)
```

The angles in the z-plane can be obtained by a combination of using the information in the document 888 and through geometry using the same principal as above.
We would like to know how the positions of the dipoles within the transducers effect the general angle given in document 888. The angle for the change due to the transducers position can be calculated by $\mathrm{Tan}[\phi] = \frac{1}{78}$ and $\mathrm{Tan}[\phi] = \frac{0.3}{78}$ respectively. Therefore the first and 4th dipole’s angle will be changed by $\approx$0.735 degree, while the 2nd and 3rd dipoles will have a change in angle given by $\approx$ 0.220 degree.

In[9]:=
```
ϕb = {15, 75, 105, 45};
(*These are just the positions of the first panels of each panel-plane*)
ϕt = {}; (*This will be a list of the ϕ - coordinates of the transducers*)
Do[Do[ϕj = ϕb[[i]] + 120 j; (*For each of the entries
    in ϕb (panel position) we have two corresponding other ones,
   which will be shifted by 120 degrees. Therefore the Do loop in j*)
    ϕ1 = ϕj - 0.735;
    ϕ2 = ϕj - 0.22;
    ϕ3 = ϕj + 0.22;
    ϕ4 = ϕj + 0.735; (*ϕ1 until ϕ4 introduce the shifting/differentiaten
    due to the positions of the dipoles in the transducers*)
    ϕtemp = {ϕ1, ϕ2, ϕ3, ϕ4};
    ϕt = Join[ϕt, ϕtemp]; (*The four values which have been calculated above will
    be added to the list, which will in the end contain all ϕ - coordinates*)
     , {j, 0, 2}], {i, 1, 4}]; (*We need the i -
 loop as well because we have the 4 different entries in ϕb*)
ϕt = Flatten[Table[ϕt, {i, 1, 9}]] π / 180; (*The Table function just creates the ϕ -
 coordinates of the transducers for the 9 stations to the right of the origin*)
ϕt = Join[ϕt, ϕt]; (*We need to duplicate the above
 list again because we have 18 stations*)
```

It is easier to add vectors in cartesian coordinates because they have unique entries. Therefore we will switch back to cartesian coordinates:

In[14]:=
```
xt = Table[rt[[i]] Cos[ϕt[[i]]], {i, 1, Length[rt]}];
yt = Table[rt[[i]] Sin[ϕt[[i]]], {i, 1, Length[rt]}];
```

The following function will calculate the magnetic field due to all transducers at a chosen point in space. It does this by first making a list of the magnetic fields due to the individual dipoles at a certain point in space and then adds all of them up.

In[16]:=
```
Btatpoint[x_, y_, z_] :=
 Apply[Plus, Table[Bcartesian[x - xt[[i]], y - yt[[i]], z - zt[[i]]], {i, 1, Length[xt]}]]
```

It probably is the case, that the magnetic feld due to the transducers is biggest somewhere in the middle of the tracker and near to one of the dipoles.
Choosing the 12 points closest to the dipoles in the $z = -5.25$ plane and calculating the magnetic field gives the following:

In[17]:=
```
Table[Btatpoint[70 Cos[ϕt[[i]]], 70 Sin[ϕt[[i]]], -5.25], {i, 1, 12}]
```

Out[17]=
```
{{0.0121592, 0.00476374, 0.0199891}, {0.0126205, 0.00392071, 0.0206443},
 {0.0128322, 0.00312911, 0.0206336}, {0.0128532, 0.00216891, 0.0199534},
 {-0.0102051, 0.00814834, 0.0199891}, {-0.00970568, 0.0089693, 0.0206443},
 {-0.00912598, 0.00954845, 0.0206336}, {-0.00830493, 0.0100467, 0.0199534},
 {-0.0019541, -0.0129121, 0.0199891}, {-0.0029148, -0.01289, 0.0206443},
 {-0.00370621, -0.0126776, 0.0206336}, {-0.00454826, -0.0122156, 0.0199534}}
```

Hence we only distort the field by a minimal amount.
To see whether the chosen points are really the ones, where the field is being distorted most, we will have a look at some 3D-plots, where the magnitude of the magnetic field in the xy - Plane is being shown.

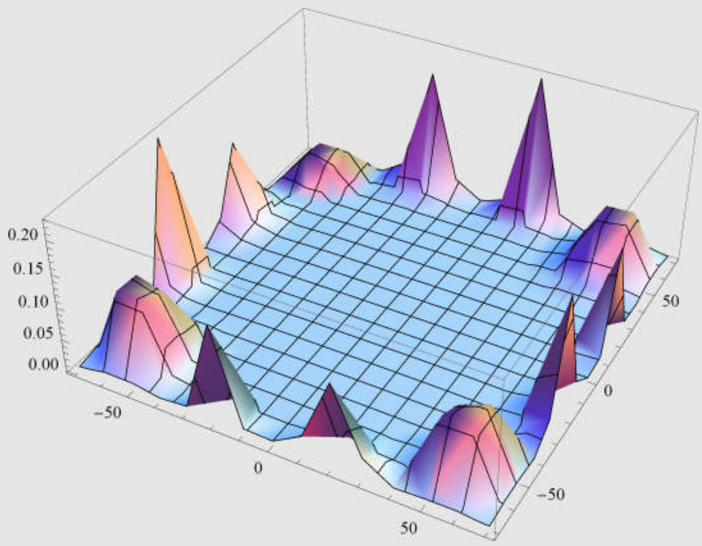
Hence we would like to calculate the magnitude of the magnetic field at a chosen point in space.
The following function exactly does this:

In[18]:=

```
Bmagnitudeatpoint[x_, y_, z_] := (s = Apply[Plus,
    Table[Bcartesian[x - xt[[i]], y - yt[[i]], z - zt[[i]]], {i, 1, Length[xt]}]];
                                         Bm = Sqrt[(s[[1]])^2 + s[[2]]^2 + s[[3]]^2])
```

Let us consider the $z = 5.25$ plane. First of all I chose the range to go from -70 to 70 for both x and y. Hence there are regions of the graph, which lie not in the concerned area, where $r \leq 70$. First of all we notice that there are 12 regions, where the field changes dramatically. This makes perfect sense, as these regions are the ones, which are close to the 12 transducer pairs in one station. Therefore our assumption, that the points, near to one of the dipoles, will experience a greater distortion is right.

```
graph1 =
 Plot3D[Bmagnitudeatpoint[x, y, 5.25], {x, -70, 70}, {y, -70, 70}, PlotRange → All]
```



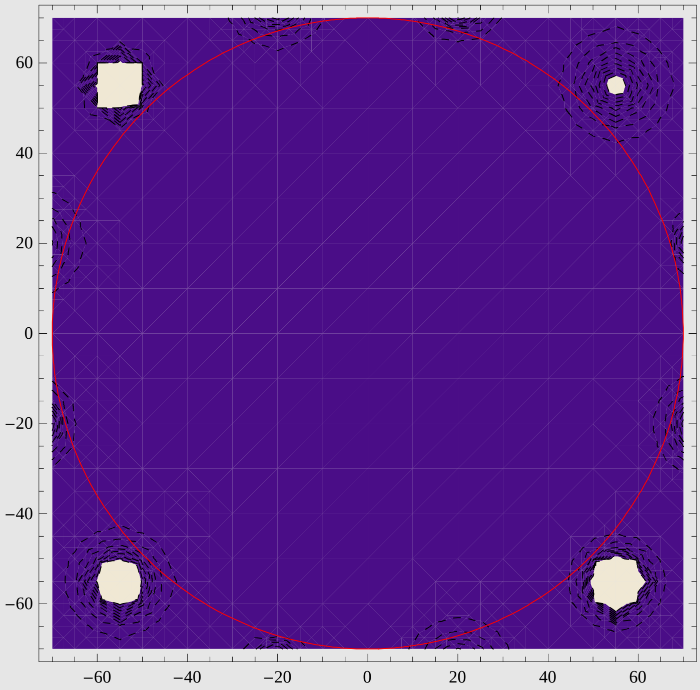
In[54]:=

```
SetDirectory[NotebookDirectory[]];
```

```
Export["graph1.jpg", graph1, "JPG"];
```

As mentioned above, not all of the regions in this plot are of concern to us, as we are only interested in the distortions in the region $r \leq 70$.
The following contour plot exactly illustrates this. The regions within the circle are the ones which interest us. We can see, that they barely come near the effects of the 12 transducer pairs.

```
graph2 = Show[ContourPlot[Bmagnitudeatpoint[x, y, 5.25], {x, -70, 70}, {y, -70, 70},
   PlotRange → All, ContourStyle → {Dashed}], ContourPlot[x^2 + y^2 == 70^2,
   {x, -70, 70}, {y, -70, 70}, PlotRange → All, ContourStyle → {Red}]]
```

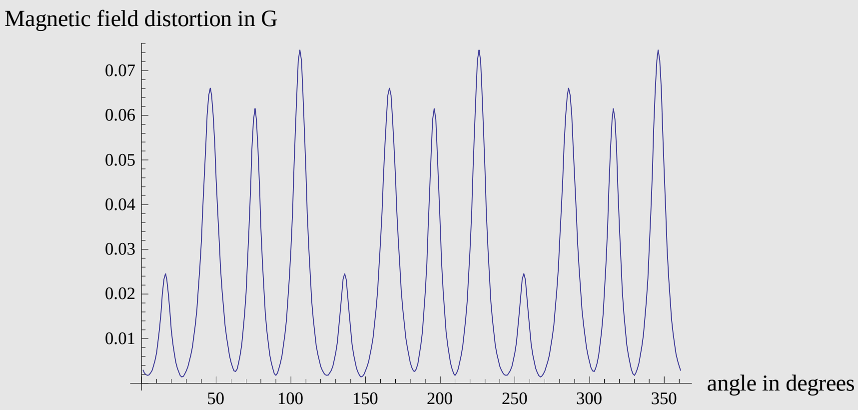



```
Export["graph2.jpg", graph2, "JPG"];
```

We now want to see what the field ditortions are on the red circle. Graph 3 will show us exactly this:

```
graph3 =
 ListPlot[Table[Bmagnitudeatpoint[70 Cos[i], 70 Sin[i], zt[[1]]], {i, 0, 2 π, π / 180}],
  AxesLabel → {Style["angle in degrees", Medium],
    Style["Magnetic field distortion in G", Medium]}, Joined → True]
```

Out[66]=




```
Export["graph4.jpg", graph3, "JPG"];
```

We see that the greates distortions are found at the points, which are closest to the 12 transducer pairs in one station.

However we need to look at all points in the tracker which are closest to the transducer pairs.
The following Do loop creates a list l, with values of field distortions at approximately 26 000 points in

space all located at $r = 70$.
This list should cover all the points with the biggest field distortions.

```
l = {};
Do[
 l = Join[Table[Bmagnitudeatpoint[70 Cos[i], 70 Sin[i], zt[[k]]], {i, 0, 2 π, π / 180}],
   l], {k, 1, 853, 12}]
```

In[45]:=
```
Max[l]
```

Out[45]=
```
0.0860757
```

The Maximum value of that list is approximately 0.086G. This is the maximum distortion by the transducers in the interested region.

Conclusion

We have shown that the distortions, due to the transducers on the magnetic field of the tracker, can be ignored as long as we position the transducers at the outer edge of our FEE space.
We can see this is of great importance as we will otherwise come near the regions, which will introduce greater distortions.