Simulation of Channeling Radiation

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Electron channeling

- Steering of electrons between planes of diamond
- **Classical Interpretation:** electron oscillates between planes (valid at high beam energies)
- **Quantum Interpretation:** electron jumps between bound states (valid at medium beam energies)

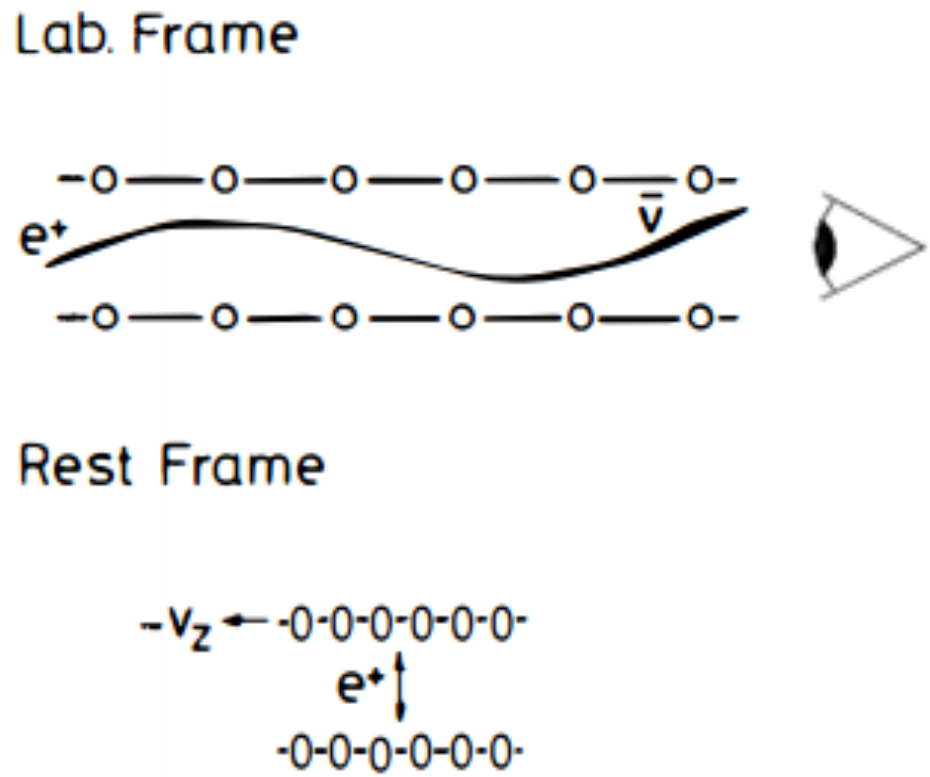

Figure: "Channeling Radiation" by Andersen, Bonderup, and Pantell

Energy levels

- Potential defined by density of atoms
- Energy levels depend on beam energy
- **Energy of emitted photons** = (Doppler-shift) x (difference between energy levels)

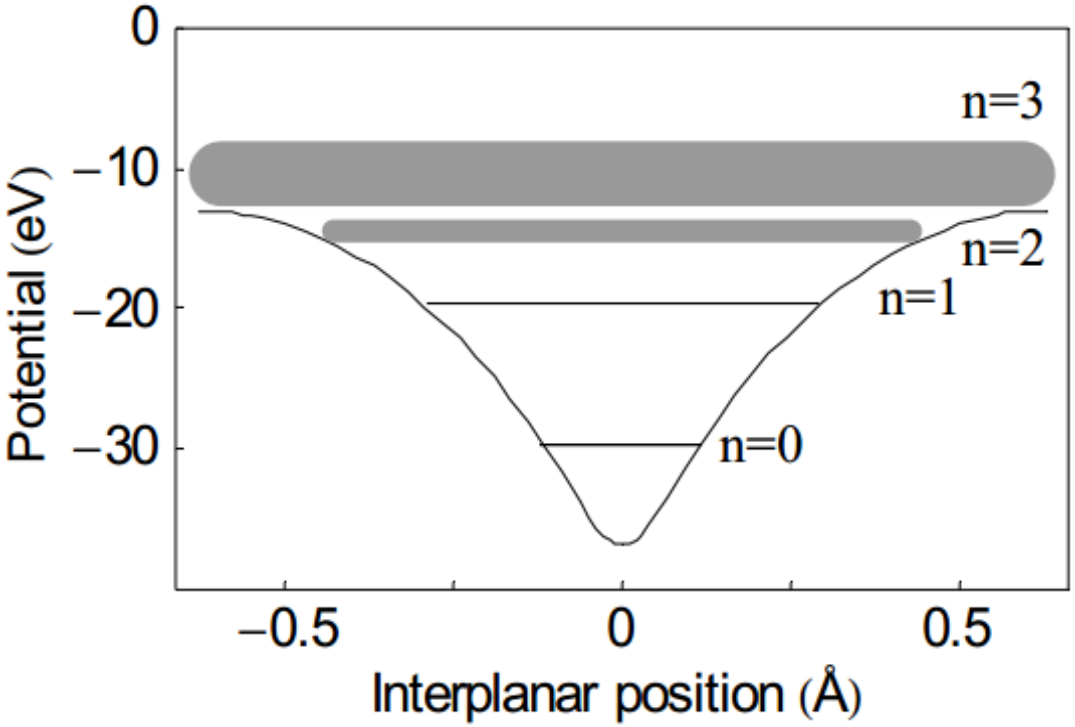

Figure: "Dissertation" by Azadegan

Why do we care?

- Features of CR:
  - nearly monochromatic
  - high intensity
- Fermilab experiments:
  - at A0 (now)
  - and ASTA (in one year)
- Possible uses:
  - medical imaging
  - military scanning

Figure: ASTA at Fermilab

Figure: "Understanding the Workings of X-rays" from Brucelashleydpm's blog

Line widths

- **Coherence length:** related to the lifetime of the channeling states
- **Multiple Scattering:** scattering of particles off crystal atoms

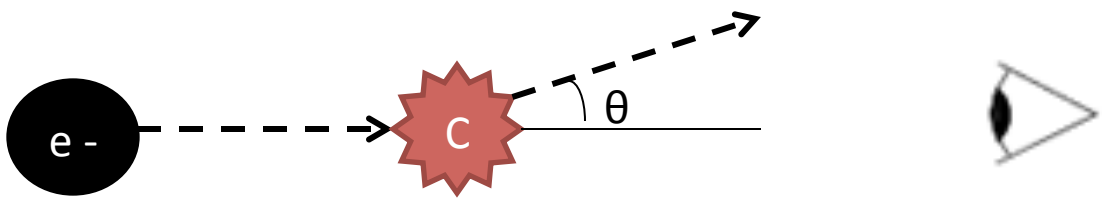

Figure: Multiple scattering of electron off Carbon atom

- **Bloch-wave:** photon energy spread due to energy bands

Photon yields (intensities)

$$\frac{d^2N(i\to f)}{d\Omega dE_\gamma} = \frac{\alpha_0\lambda_c^2}{\pi\hbar c}E_0(i\to f)|\langle\psi_f|\frac{d}{dx}|\psi_i\rangle|^2\int_0^L \exp\left(-\mu(E_\gamma)(L-z)\right)P_i(z)dz$$
$$\times\,\pi^{-3/2}\int_0^\infty \frac{t^{-1/2}(\Gamma_T/2)(1+2\alpha^2t)e^{-t}}{(E_\gamma(1+2\alpha^2t)-E_0)^2+0.25(1+2\alpha^2t)^2\Gamma_T^2}dt$$

- Peak photon energy
- Dipole matrix element: strength of transition
- Population dynamics
- Line shape: convolution of Lorentzian and Gaussian

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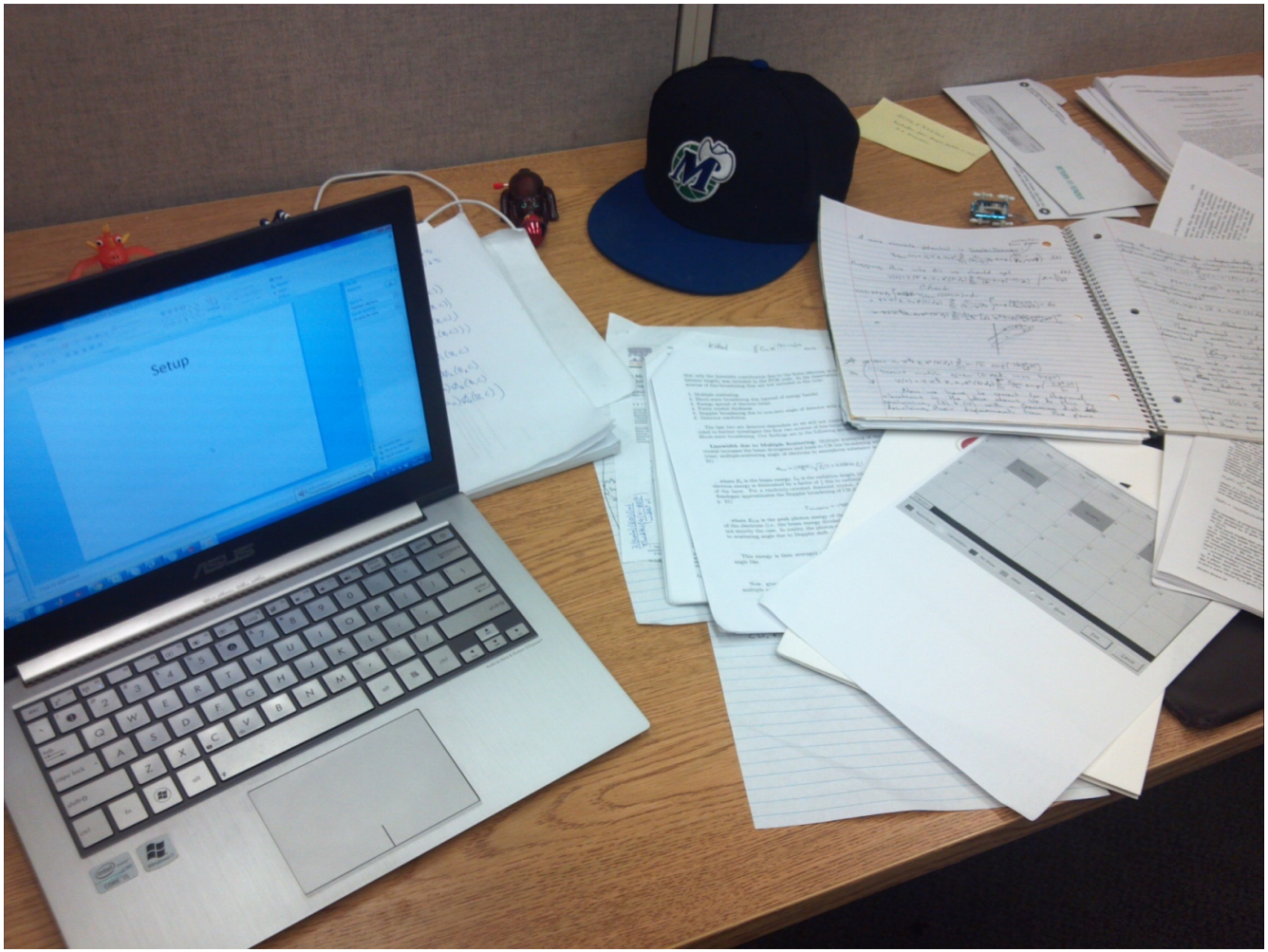
Experimental setup



PCR code

- Simulates electron channeling using quantum interpretation
- Generates spectrum:

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Figure: 14.6 MeV electrons channeled in (110) plane of diamond (100 microns thick)

Peak energies

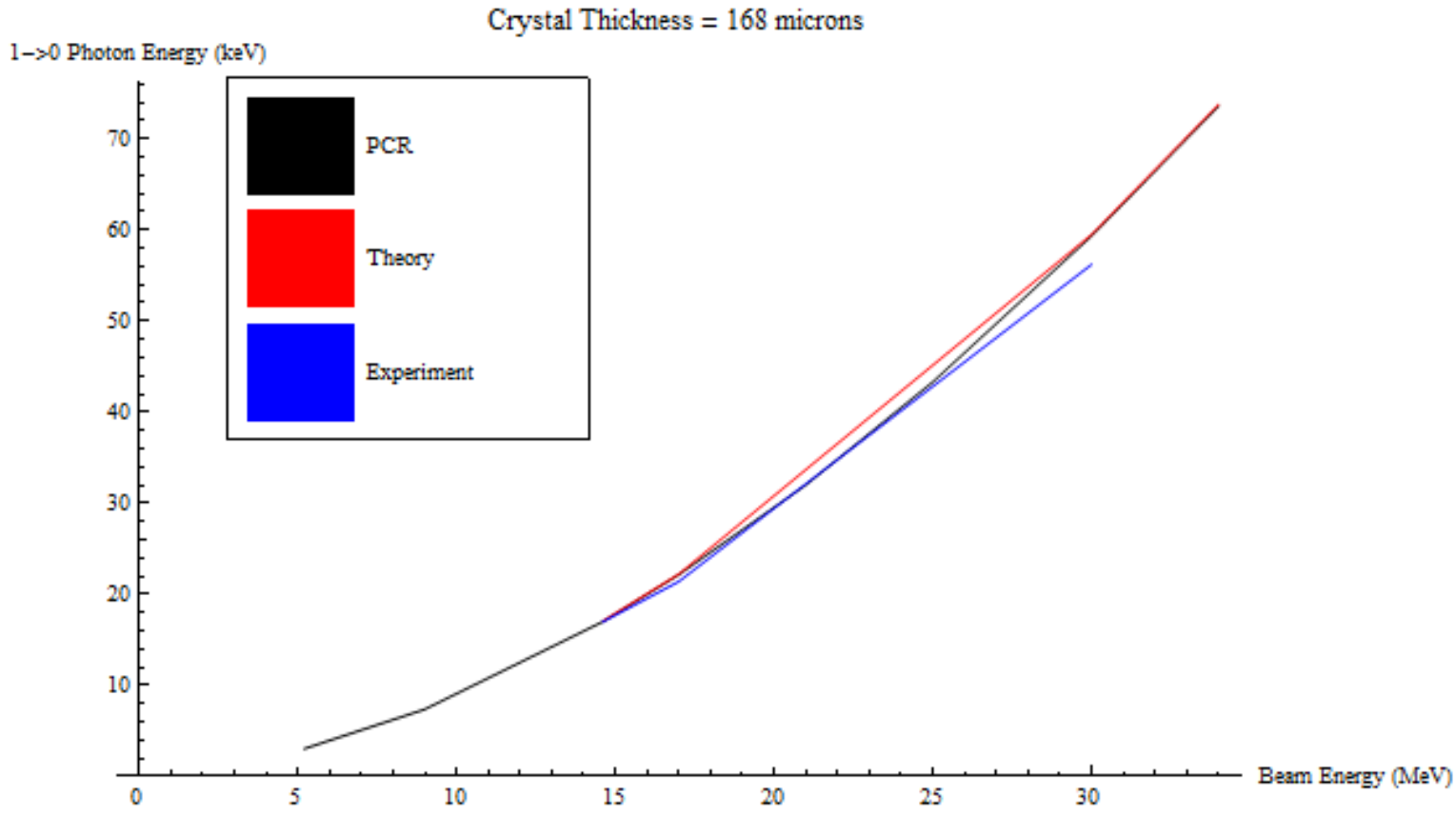

Figure: Peak energies of PCR, theory, and experiment for electrons channeled in (110) plane of diamond (168 microns thick).

Problem with PCR: line widths

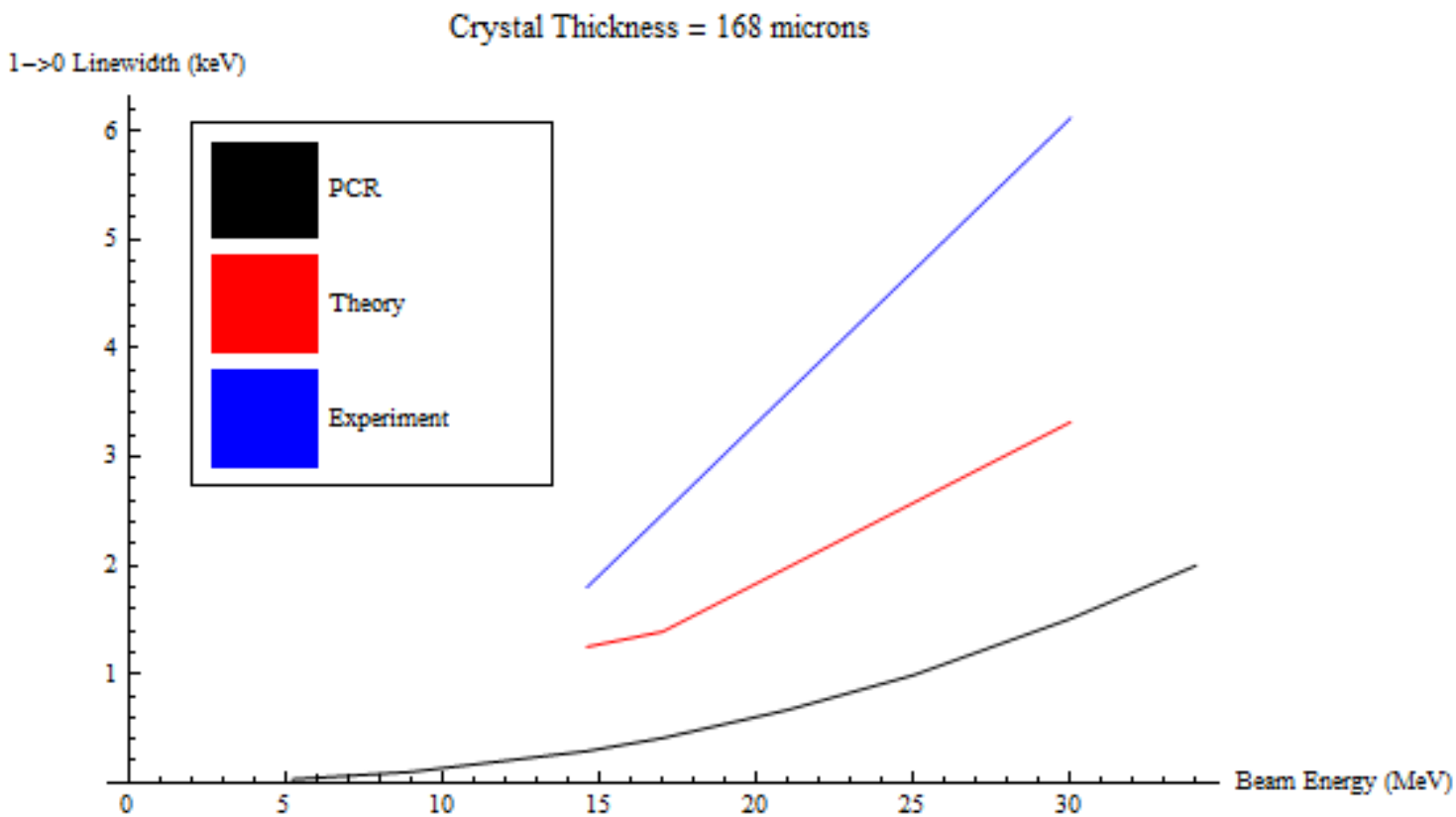

Figure: Line width of PCR, theory, and experiment for electrons channeled in (110) plane of diamond (168 microns thick).

Solution: line widths

- PCR only includes coherence length!
- Add multiple scattering and Bloch-wave
- Actually improved on multiple scattering from theory

Problem with PCR: photon yields

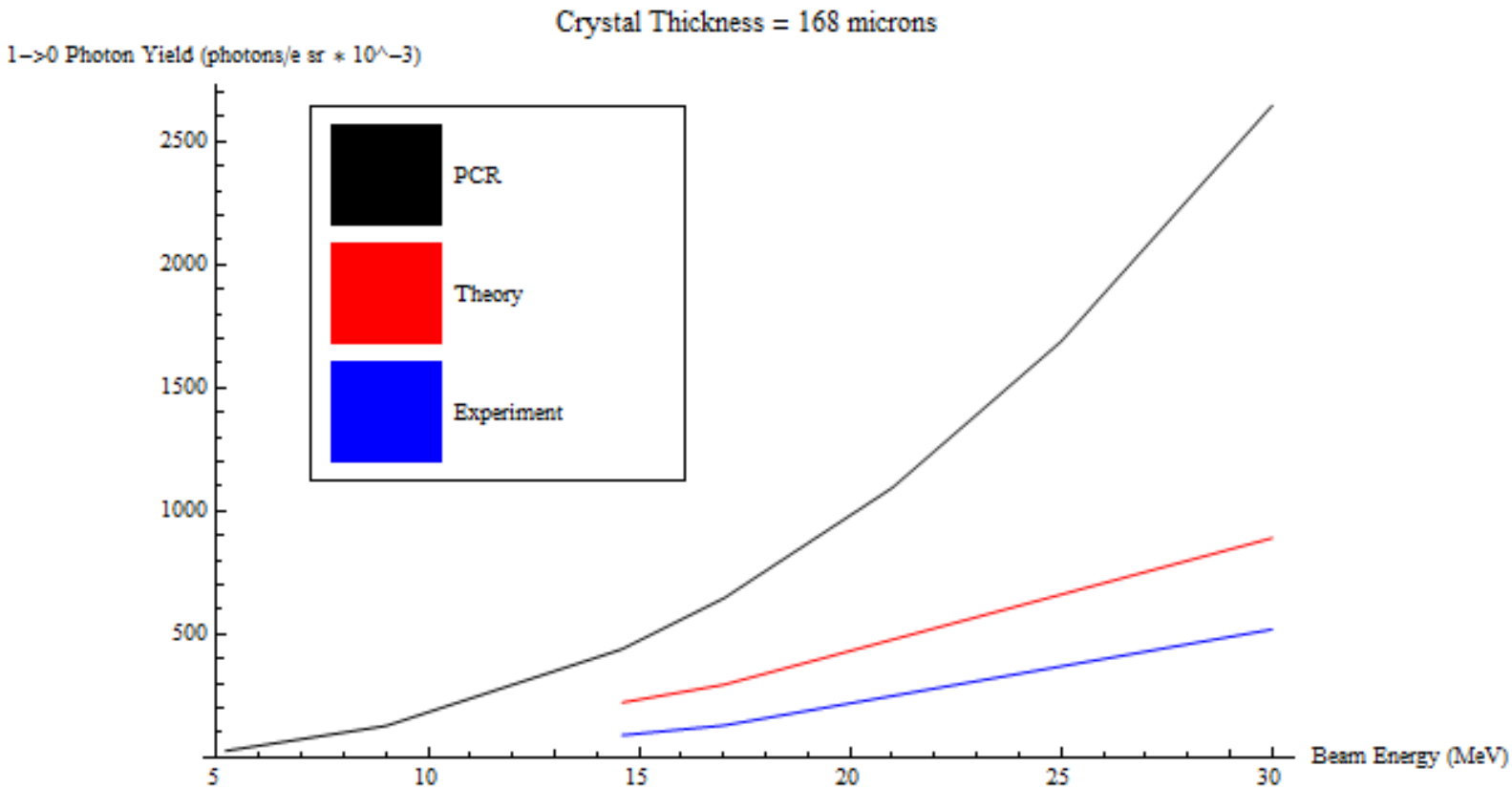

Figure: Photon yield of PCR, theory, and experiment for electrons channeled in (110) plane of diamond (168 microns thick).

Solution: photon yields

$$\frac{d^2N(i\to f)}{d\Omega dE_\gamma} = \frac{\alpha_0\lambda_c^2}{\pi\hbar c}E_0(i\to f)|\langle\psi_f|\frac{d}{dx}|\psi_i\rangle|^2\int_0^L \exp\left(-\mu(E_\gamma)(L-z)\right)P_i(z)dz$$
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- Change line shape to convolution (5% drop)
- Add photon self-absorption (3% drop)
- Include dechanneling of electrons!

What to do now?

- Put it all together!
- Analysis of new code
- Include realistic beam
- Include detector effects

Thank ya!